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## Erratum

# Erratum to “On openness and surjectivity of lifted frame homomorphisms” [Topology Appl. 157 (2010) 2159–2171]

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## ABSTRACT

We address and correct an oversight that we have identified in Lemma 3.2 of Dube and Naidoo (2010) [2] and provide counterexamples to the original assertions.

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We have come to notice that the assertion in Lemma 3.2 is incorrect. The flaw in the “proof” is that, from the equality

$$\gamma_M \cdot (\gamma_M)_* \cdot h = \gamma_M \cdot h^\gamma \cdot (\gamma_L)_*,$$

we asserted that  $\gamma_M$ , being a dense frame homomorphism, is left-cancellable in the equation. Indeed, dense frame homomorphisms are monic in the category **RegFrm**, so that they are left-cancellable whenever composed with morphisms in **RegFrm**. Now, neither  $(\gamma_M)_* \cdot h$  nor  $h^\gamma \cdot (\gamma_L)_*$  need be a morphism in this category.

This error impacts the paper in the following way. Corollary 3.4 and Proposition 3.5 are then false. However, the forward implications in these results are correct. Here is a verification.

Let  $\gamma$  and  $\delta$  be as in Corollary 3.4. For brevity, say  $\gamma$  dominates  $\delta$  if, as in the hypothesis of Corollary 3.4, for every  $K \in \mathbf{CRegFrm}$  there is a dense onto frame homomorphism  $j_K: \gamma K \rightarrow \delta K$  such that  $\gamma_K = \delta_K \cdot j_K$ . That then implies the triangles, the trapezoids and the outer square in the diagram

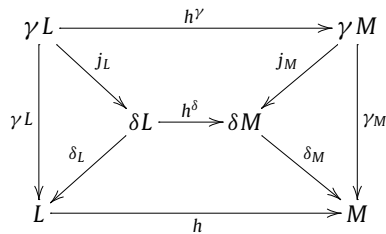
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all commute. The correct statement of Corollary 3.4 should then be:

Let  $\gamma$  and  $\delta$  be  $\mathbf{C}$ -fication functors with  $\gamma$  dominating  $\delta$ . If  $h : L \rightarrow M$  is a  $\gamma$ -map, then it is a  $\delta$ -map.

**Proof.** Since the upper trapezoid in the diagram above commutes,  $j_M \cdot h^\gamma = h^\delta \cdot j_L$ , so that, in light of  $j_L \cdot (j_L)_* = \text{id}_{\delta L}$ , as  $j_L$  is onto, we have

$$h^\delta = j_M \cdot h^\gamma \cdot (j_L)_*. \quad (1)$$

Since  $\gamma_L = \delta_L \cdot j_L$ , we have

$$(\gamma_L)_* = (j_L)_* \cdot (\delta_L)_*. \quad (2)$$

Similarly,  $(\gamma_M)_* = (j_M)_* \cdot (\delta_M)_*$ , which implies

$$j_M \cdot (\gamma_M)_* = j_M \cdot (j_M)_* \cdot (\delta_M)_* = (\delta_M)_*. \quad (3)$$

Now, from (1),

$$\begin{aligned} h^\delta \cdot (\delta_L)_* &= j_M \cdot h^\gamma \cdot (j_L)_* \cdot (\delta_L)_* \\ &= j_M \cdot h^\gamma \cdot (\gamma_L)_* \quad \text{by (2)} \\ &= j_M \cdot (\gamma_M)_* \cdot h \quad \text{since } h \text{ is a } \gamma\text{-map} \\ &= (\delta_M)_* \cdot h \quad \text{by (3)}. \end{aligned}$$

Therefore  $h$  is a  $\delta$ -map.  $\square$

Note then that, in view of  $\beta$  dominating  $\lambda$ , and  $\lambda$  dominating  $\nu$ , as the diagrams in the “proof” of Proposition 3.5 attest, the statement of Proposition 3.5 can be corrected to state:

Every  $\beta$ -map is a  $\lambda$ -map, and every  $\lambda$ -map is an  $\nu$ -map.

Consequently, recalling that our definition on page 2161 of  $N$ -map is precisely that of  $\beta$ -map, the statements of the main result (Proposition 3.8) and its corollary are correct.

**Notes.** In view of the above, the term “ $N$ -map” in the last sentence of Remark 3.7 should be replaced with “ $\lambda$ -map”. Also, in the proof of the implication  $(\Leftarrow)$  of Proposition 3.12, the equality  $h = \gamma_M \cdot h^\gamma \cdot (\gamma_L)_*$  was implied to be valid by virtue of  $h$  being a  $\gamma$ -map. Actually, this holds for any homomorphism  $h$ . Thus, the more accurate statement of the proposition (with exactly the same proof, *mutatis mutandis*) should be:

For any homomorphism  $h : L \rightarrow M$ , if  $h^\gamma$  is nearly open, then  $h$  is nearly open. Conversely, if  $h$  is a  $\gamma$ -map and is nearly open, then  $h^\gamma$  is nearly open.

Lastly, we give an example of a  $\lambda$ -map which is not a  $\beta$ -map, and an example of an  $\nu$ -map which is not a  $\lambda$ -map. Observe that any homomorphism  $h : L \rightarrow M$  is:

- a  $\beta$ -map if both  $L$  and  $M$  are compact,
- a  $\lambda$ -map if both  $L$  and  $M$  are Lindelöf,
- a  $\nu$ -map if both  $L$  and  $M$  are realcompact.

It is shown in [1, Proposition 6.2] that a  $\beta$ -map with a normal codomain is closed.

**Example 1** (*A  $\lambda$ -map which is not a  $\beta$ -map*). Let  $L$  be a Lindelöf frame which is not compact. Then the join map  $\beta L \rightarrow L$  is a  $\lambda$ -map. It is however not a  $\beta$ -map. Indeed, if it were, then it would be closed, and hence, being dense, it would be codense, and hence one-one, and hence an isomorphism – which it is not.

Next, notice that

*a homomorphism  $h : L \rightarrow M$  is a  $\lambda$ -map if and only if for any  $z \in \text{Coz } M$  and  $a \in L$ ,  $z \leq h(a)$  implies  $z \leq h(c)$  for some  $c \leq a$  in  $\text{Coz } L$ .*

So if the join map  $\lambda L \rightarrow L$  is a  $\lambda$ -map, then it must be codense, meaning that it maps only the top to the top. Indeed, if  $\bigvee J = 1$  for  $J \in \lambda L$ , then, being a  $\lambda$ -map, there exists  $c \in \text{Coz } L$  such that  $[c] \subseteq J$  and  $1 \leq \bigvee [c] = c$ , implying  $J = 1_{\lambda L}$ .

**Example 2** (*An  $\nu$ -map which is not a  $\lambda$ -map*). Let  $L$  be a realcompact frame which is not Lindelöf. Then the join map  $\lambda L \rightarrow L$  is an  $\nu$ -map since its domain and codomain are realcompact. If it were a  $\lambda$ -map, then it would be codense and hence an isomorphism – which it is not.

## References

- [1] T. Dube, Remote points and the like in pointfree topology, Acta Math. Hungar. 123 (2009) 203–222.
- [2] T. Dube, I. Naidoo, On openness and surjectivity of lifted frame homomorphisms, Topology Appl. 157 (2010) 2159–2171.